

Coulomb interaction and electron-hole asymmetry in cyclotron resonance of bilayer graphene in high magnetic field

V. E. Bisti^{1,2}, N. N. Kirova^{2*}

¹*Institute of Solid State Physics, Russian Academy of Sciences, 142432, Chernogolovka, Russia,* ²*LPS, CNRS and Université Paris-Sud, 91405, Orsay Cedex, France*

(Dated: January 12, 2013)

Inter-Landau-level transitions in the bilayer graphene in high perpendicular magnetic field at the filling-factor $\nu = 0$ have been studied. The next-nearest-neighbor transitions, energy difference between dimer and non-dimer sites, and layer asymmetry are included. The influence of Coulomb interaction is taken into account. The magnetoplasmon excitations in bilayer graphene at small momenta are considered within the Hartree-Fock approximation. The asymmetry in cyclotron resonance of clean bilayer graphene is shown to depend on magnetic field. At lower magnetic fields the energy splitting in the spectrum is due to electron-hole one-particle asymmetry while at higher magnetic fields it is due to Coulomb interaction. For the fully symmetric case with half-filled zero-energy levels the energy splitting proportional to the energy of Coulomb interaction is found both for bilayer and monolayer graphene.

PACS numbers: 73.22.Lp, 73.43.Lp, 76.40.+b, 78.66.Tr

I. INTRODUCTION

Recent experimental progress has allowed the fabrication and study of monolayer and bilayer graphene. The electronic band structure of these objects is gapless and has a chirality¹. The monolayer has Dirac-type spectrum with linear dispersion and chirality exhibiting Berry phase π . The bilayer graphene is the unique object which combines the parabolic dispersion law of quasiparticles near the zero energy point with their chirality exhibiting Berry phase 2π . This picture is obtained with the tight-binding Hamiltonian for electrons taking into account only nearest-neighbor transitions; the one-electron spectrum is symmetric around zero energy. Taking into account next-nearest-neighbor transitions results in the asymmetry of electron spectrum around zero-energy point². Some experimental data demonstrate electron-hole asymmetry in cyclotron resonance spectra of clean bilayer graphene³⁻⁷.

One-particle Landau levels in the bilayer graphene at high magnetic fields have been considered in the works^{8,9} taking into account only nearest-neighbor transitions. In magnetic field there is a two-fold degenerate zero-energy Landau level incorporating two different orbital states with the same energy. Taking into account spin and valley degeneracies, the zero-energy Landau level in a bilayer is eight-fold degenerate. For the bilayer with small asymmetry there are four weakly split two-fold degenerate levels near zero energy. The valley and orbital degeneracies are lifted, but the electron-hole symmetry is preserved.

The near-zero-levels are strongly influenced by Coulomb electron-electron interaction. The electron-electron interaction is an important problem in the experimental study of cyclotron resonance in monolayer^{10,11}, bilayer^{3,12} and multilayer^{7,13-17} graphene, exhibiting some properties of a monolayer and bilayer. Interaction-induced shift of the cyclotron resonance as a function of the filling-factor in bilayer graphene was studied in

Ref.¹², the symmetry breaking in the zero-energy Landau level in bilayer graphene is demonstrated in¹⁸.

The charge-density excitations at small momenta were considered theoretically within the Hartree-Fock approximation for monolayer graphene^{19,20} and for bilayer²¹. In the works²²⁻²⁴ electromagnetic response in graphene was calculated numerically in the RPA approximation for wide range of excitation momenta. The spin-flip excitations and spin-waves in graphene were studied in Ref.²⁵. In the works^{26,27} intra-Landau level transitions were considered. The many-body corrections obtained within the renormalization method, including weak electron-hole asymmetry, and the attempts to explain sharp transition from square to linear dispersion regime were reported in Refs.²⁸⁻³⁰. In the works¹⁹⁻²¹ Coulomb interaction was shown to conserve electron-hole symmetry for excitations.

In the present paper the inter-Landau-level transitions in the bilayer graphene in high perpendicular magnetic field at the filling-factor $\nu = 0$ are studied. The novelty of this work is that the electron-hole asymmetry and Coulomb interaction are included into consideration simultaneously. Special attention is given to the difference in the cyclotron transition energies for two valleys under different conditions. First, the one-particle Hamiltonian and Landau levels are considered taking into account the electron-hole asymmetry due to next-nearest-neighbor transitions and energy difference between the dimer and non-dimer sites and the small energy difference between the layers due to external potential. Then, the influence of Coulomb interaction is included. The charge-density excitations (magnetoexcitons) at small momenta are considered within the Hartree-Fock approximation in the case of clean (neutral) bilayer graphene with filling-factor $\nu = 0$. The energies of excitations are shown to be different in the two valleys, and the origin of this difference depends on magnetic field. At lower magnetic fields the energy splitting is due to electron-hole one-particle

asymmetry while at higher magnetic fields the energy splitting in the spectrum is due to Coulomb interaction. Next, the results are discussed in connection with experimental possibility to observe the influence of Coulomb interaction, and the comparison with other theoretical works is presented.

II. HAMILTONIAN AND LANDAU LEVELS OF BILAYER GRAPHENE

The bilayer is modelled as two coupled hexagonal lattices with inequivalent sites (A1, B1) and (A2, B2) in the first and second graphene layers, respectively, arranged according to Bernal (A2-B1) stacking. In the tight-binding model the energy states of electrons in (A1-B2) dimer in the vicinity of zero-energy point are conveniently described by an effective two-component Hamiltonian^{2,8,9} that operates in the space of wave functions $\Psi = (\psi_{A1}, \psi_{B2})$ in the valley K and of $\Psi = (\psi_{B2}, \psi_{A1})$ in the valley K' . The asymmetry between on-site energies in the two layers U arising from the influence of external gates or a doping effect, the next-nearest-neighbor transitions and the difference between on-site energies of dimer and non-dimer sites $\tilde{\Delta}$ are taken into account:

$$H = H_0 + H_1 + H_2 \quad (1)$$

$$H_0 = -\frac{1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix} \quad (2)$$

$$H_1 = \frac{\xi U}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \frac{1}{2m} \frac{\xi U}{\gamma_1} \begin{pmatrix} \pi^+ \pi & 0 \\ 0 & -\pi \pi^+ \end{pmatrix} \quad (3)$$

$$H_2 = \frac{1}{2m} \left(\frac{2\gamma_4}{\gamma_0} - \frac{\tilde{\Delta}}{\gamma_1} \right) \begin{pmatrix} \pi^+ \pi & 0 \\ 0 & \pi \pi^+ \end{pmatrix} + \frac{\tilde{\Delta}}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (4)$$

where $\pi = \hbar k_x + i\hbar k_y$, $\pi^+ = \hbar k_x - i\hbar k_y$ are the complex momentum operators, \mathbf{k} is the wave vector measured from the center of the valley, ξ is the valley index, $\xi = 1$ in the valley K , $\xi = -1$ in the valley K' , γ_0 is the intra-layer A-B coupling parameter, γ_1 is the inter-layer A2-B1 coupling parameter, $m = \gamma_1/2v^2$ is the effective mass for bilayer graphene, $v = \frac{\sqrt{3}}{2\hbar} a\gamma_0$, a is the lattice constant.

The parameter γ_4 describes the next-nearest-neighbor transitions (A1-A2 and B1-B2 interlayer hopping), $\gamma_1 = 0.1\gamma_0$, $\gamma_4 = 0.05\gamma_0$ (see^{5,6}).

H_0 is the basic term yielding a parabolic spectrum with the effective mass m , and its quasiparticles are chiral with the degree of chirality related to Berry phase 2π .

H_1 describes the layer asymmetry, leading to the opening of a gap $\sim U$ in the spectrum.

H_2 is due to the next-nearest-neighbor transitions and the difference between on-site energies of dimer and

non-dimer sites. The first term is responsible for the electron-hole asymmetry in the spectrum around zero-energy point. The second term due to $\tilde{\Delta}$ results in the shift of the single-energy spectrum as a whole. This term is unimportant for our considerations and will be omitted later.

The two-component Hamiltonian is applicable if the considered electron energy ε is within the energy range of $|\varepsilon| < \frac{1}{4}\gamma_1$. The weak asymmetry means that $U/\gamma_1 \ll 1$, $\Delta/\gamma_1 \ll 1$, $\gamma_4/\gamma_0 \ll 1$.

In the perpendicular magnetic field B the energy spectrum of Landau levels $E_{n\xi}$ ($n = 0, 1, \pm N$, $N = 0, 1, 2, \dots$) and corresponding two-component wave functions $\Psi_{n\xi}$ are found from the Hamiltonian H (1) using the Landau gauge $\mathbf{A} = (0, Bx)$ and raising and lowering operators $a^+ = l_B \pi^+ / \sqrt{2}$ and $a = l_B \pi / \sqrt{2}$, as in the work⁹. The magnetic length l_B and the cyclotron frequency ω_c are defined as usual: $l_B = \sqrt{\hbar/eB}$, $\omega_c = eB/m$, e is the electron charge. The basis consisting of the wave functions describing the states in the ordinary two-dimensional electron gas $\phi_{Nk} = e^{iky} \phi_{Nk}(x)$ is used, where k is the parameter which labels degenerate states within one Landau level in Landau gauge.

$$E_0(\xi) = \frac{1}{2}\xi U, \quad E_1(\xi) = \frac{1}{2}\xi U - \xi\delta + \left(\frac{2\gamma_4}{\gamma_0} - \frac{\tilde{\Delta}}{2\gamma_1}\right)\hbar\omega_c \quad (5)$$

$$E_{\pm N}(\xi) = \pm\hbar\omega_c\sqrt{N(N-1)} - \frac{1}{2}\xi\delta + \left(\frac{\gamma_4}{\gamma_0} - \frac{\tilde{\Delta}}{2\gamma_1}\right)\hbar\omega_c(2N-1) \quad (6)$$

where $\delta = U\hbar\omega_c/\gamma_1$.

$$\Psi_{0k} = (\phi_{0k}, 0), \quad \Psi_{1k} = (\phi_{1k}, 0),$$

$$\Psi_{n\xi} = (a_{n\xi}\phi_{Nk}, b_{n\xi}\phi_{N-2,k}) \quad (7)$$

The coefficients $a_{n\xi}$ and $b_{n\xi}$ are the eigenvector components.

$$a_{n\xi} = 1/\sqrt{1+D_{n\xi}}, \quad b_{n\xi} = D_{n\xi}/\sqrt{1+D_{n\xi}}$$

$$D_{n\xi} = \frac{E_{n\xi} - \xi U/2 + \xi N\delta - \left(\frac{\gamma_4}{\gamma_0} - \frac{\tilde{\Delta}}{2\gamma_1}\right)\hbar\omega_c(2N-1)}{\hbar\omega_c\sqrt{N(N-1)}} \quad (8)$$

Without any asymmetry in the zero approximation $a_{\pm N, \xi} = 1/\sqrt{2}$, $b_{\pm N, \xi} = \pm 1/\sqrt{2}$.

The asymmetry splits the zero-energy Landau level degenerate in valleys and orbital momenta into four levels. Energy levels for $N \geq 2$ are weakly split in valleys.

Note that the spectrum of high-energy LLs is applicable for the fields and levels satisfying the condition $\hbar\omega_c\sqrt{N(N-1)} < \gamma_1/4$. For $\gamma_1 = 0.39\text{eV}$ this inequality yields $B < 50T$ for $N = 2$. For higher fields or higher levels the full four-band Hamiltonian has to be used to determine the exact LL spectrum^{31,32}.

The Zeeman splitting is omitted, and all levels are doubly degenerate in spin. Although in graphite the electron g -factor is not small ($g = 2$), a very light effective mass $m \approx 0.054$ in the bilayer determines a small ratio between the Zeeman energy and LL splitting $\varepsilon_Z/\hbar\omega_c \sim 0.05^9$. Trigonal warping coming from $\gamma_3 = \gamma_{A1-B2} \ll \gamma_1$ is not included.

III. COULOMB INTERACTION AND MAGNETOEXCITATIONS

The total Hamiltonian of the many-body system in the perpendicular magnetic field with the Coulomb interaction is

$$\hat{H} = \sum E_{n\xi} a_{\lambda\xi\sigma}^+ a_{\lambda\xi\sigma} + H_{int} \quad (9)$$

where $a_{\lambda\xi\sigma}^+$ and $a_{\lambda\xi\sigma}$ are the one-particle creation and annihilation operators; $\lambda = (n, k)$, $n = 0, 1, \pm N$ indicates the Landau level; k is the parameter which labels degenerate states within one Landau level in Landau gauge; ξ and σ are valley and spin indexes.

$$H_{int} = \frac{1}{2} \sum V_{\lambda 3, \lambda 4}^{\lambda 1; \lambda 2} a_{\lambda 4 \xi \sigma}^+ a_{\lambda 3 \xi' \sigma'}^+ a_{\lambda 2 \xi' \sigma'} a_{\lambda 1 \xi \sigma} \quad (10)$$

The Coulomb interaction conserves spin and valley indexes; $k1 + k2 = k3 + k4$.

The matrix elements for Coulomb interaction are found using the two-component wave functions (7), in analogy with calculations for monolayer graphene¹⁹.

$$V_{\lambda 3, \lambda 4}^{\lambda 1; \lambda 2} = V(q) e^{iq_x(k_1 - k_2 - q_y)} \tilde{J}_{n_4, n_1}(q) \tilde{J}_{n_3, n_2}(-q) \quad (11)$$

$$\tilde{J}_{m, n}(q) = a_n^* a_m J_{|m||n|}(\mathbf{q}) + b_n^* b_m J_{|m|-2, |n|-2}(\mathbf{q}) \quad (12)$$

$$J_{m, n}(\mathbf{q}) = \left(\frac{n!}{m!}\right)^{1/2} e^{-\frac{q^2}{4}} \left(\frac{q_y + iq_x}{\sqrt{2}}\right)^{m-n} L_n^{m-n}\left(\frac{q^2}{2}\right), \quad (13)$$

where $V(q) = \frac{2\pi}{\varepsilon q}$, $\mathbf{q} = (q_x, q_y)$, $k4 = k1 + q_y$, $k3 = k2 - q_y$. $J_{m, n}(\mathbf{q}) = J_{n, m}^*(-\mathbf{q})$ ($m > n$); L_n^{m-n} are Laguerre polynomials.

In Eq. (10) the summation is over the ensemble $n1, n2, n3, n4$, $k1, k2$, both spins, both valleys and the wave vector $\mathbf{q} = (q_x, q_y)$.

In this work only the charge-density-excitations (transitions without changing the electron spin) are studied, valley and spin indexes (ξ, σ) are not changed. Corresponding operators for excitations (n, n') from the level n to the level n' with the momentum K are

$$Q_{n, n'; \xi \sigma}^+(K) = \sum_k a_{\lambda' \xi \sigma}^+ a_{\lambda \xi \sigma} \quad (14)$$

where $\lambda = (n, k)$, $\lambda' = (n', k + K)$. It is assumed that the magnetic field is high which means that $E_c \ll \hbar\omega_c$, where

E_c is the typical Coulomb energy: $E_c = e^2/\varepsilon l_B$. The momentum of excitation is small: $K l_B \ll 1$. The problem is considered in the way analogous to that employed in³³ for the two-dimensional electron gas with quadratic dispersion and in^{19,20,25} for monolayer graphene systems. The time-dependent Hartree-Fock approximation is used. The Hartree-Fock approach assumes that there is a small parameter $E_c/\Delta E_{nn'}(\xi) \ll 1$, where $\Delta E_{nn'}(\xi)$ is the transition energy without interaction.

$$\Delta E_{nn'}(\xi) = E_{n'}(\xi) - E_n(\xi) \quad (15)$$

For monolayer graphene the ratio $\Delta E_{10}/E_c = 2.77$ (see¹⁹) and it does not depend on the magnetic field strength due to linear dispersion. As stated in²⁵, the method works better for spin-flip excitations. For bilayer graphene $E_c = 10\sqrt{B}$, $\hbar\omega_c = 2.2B$ (see⁸) and the ratio $\hbar\omega_c/E_c = 0.22B^{1/2}$ for $\epsilon = 5$. For the first high-energy transition $E_{12} \simeq \sqrt{2}\hbar\omega_c$, and therefore for $B = 40T$ the ratio $E_{12}/E_c \simeq 2$. We do not consider with this method the low-energy transitions between Landau levels 0 and 1 with energies close to zero.

The excitation energy $\tilde{E}_{n, n'; \xi \sigma}$ consists of noninteracting and Coulomb parts:

$$\tilde{E}_{n, n'; \xi \sigma} = \Delta E_{nn'}(\xi) + E_{nn'; \xi \sigma}^c \quad (16)$$

Coulomb part $E_{nn'}^c$ is represented by the three terms: “excitonic” $E_{nn'}^{ex}$ part due to direct interaction of the electron at the level n' and the hole at the level n , exchange self-energy $\Sigma_{n\xi\sigma}$ and $\Sigma_{n'\xi\sigma}$ corrections to the one-electron Landau level energies and “depolarization” shift which is given in the random phase approximation (RPA). The RPA part is proportional to K , and is important for dispersion (Ref.²⁰). However, we are interested in the terms responsible for intervalley splitting which are constants independent of K . Therefore the RPA part can be omitted. This restriction enables to consider excitations with different (ξ, σ) independently.

$$E_{n, n'; \xi \sigma}^c = E_{n, n'}^{ex} + \Sigma_{n'\xi\sigma} - \Sigma_{n\xi\sigma} \quad (17)$$

As for monolayer graphene, there is the problem of divergency of exchange self-energy Σ_n due to summation over all filled LLs. The spectrum of monolayer and bilayer graphene described by the model Hamiltonian is unbounded both from above and below. This fact is physically artificial. In²⁰ the cut-off value on energy or number of LL was defined. In¹⁹ the semi-empirical way was used to treat the problem. The electron-electron interaction parameter is fitted for one type of transition to experimental data. For the bilayer graphene the area of parabolic dispersion is less than required cut-off value, and it is necessary to consider the four-band Hamiltonian²⁸⁻³⁰. In this work some general rules are presented allowing to conclude where the Coulomb part can be seen.

The interlayer electron transitions from the top filled (fully or partially) to the next free (fully or partially)

Landau levels with energies nearly ω_c are considered. The selection rules for these transitions are $\Delta N = 1$. The case of filling-factor $\nu = 0$ is considered. The Fermi level is equal to zero. This filling-factor means the absence of charge density (the absence of free carriers or the equal amount of holes and electrons). Different possible ground states in magnetic field and different cyclotron transitions may correspond to this filling.

A. The asymmetric bilayer without e-h asymmetry.

Let $U > 0$. In this case we have filling-factor $\nu = 4$ for the electrons in one valley and $\nu = 4$ for the holes in another valley. For the valley with $\xi = 1$ there is the top filled LL with $n = -2$ and the transition $(-2,1)$, and for the valley with $\xi = -1$ there are the top filled 0 and 1 LLs and the transition $(1,2)$. Including spin there are two transitions of each type. The noninteracting part is the same for both types of transitions:

$$\Delta E_{-2,1}(1) = \Delta E_{1,2}(-1) = \omega_c \sqrt{2} + \frac{1}{2}|U - \delta| \quad (18)$$

Note that the electron-hole symmetry of one-particle Hamiltonian leads to the fact that $(-2,1)$ and $(1,2)$ transitions are really the same and have the same energy. $(-2,1)$ in electron representation is $(1,2)$ in hole representation. Taking into account spin degeneracy we have four transitions with equal energies. Since all types of asymmetry and electron-electron interaction are considered as small perturbations, the many-body corrections can be calculated with symmetric wave functions.

For $\nu = 0$ integer filling, labelled $0I$,

$$E_{ex,0I}^{(1,2)} = -\frac{1}{(2\pi)^2} \int d\mathbf{q} V(q) \tilde{J}_{22}(\mathbf{q}) \tilde{J}_{1,1}(-\mathbf{q}) = \frac{1}{2(2\pi)^2} \int d\mathbf{q} V(q) (J_{22}(q) + J_{00}(q)) J_{11}(q) \quad (19)$$

$$\begin{aligned} (\Sigma_2 - \Sigma_1)_{0I} &= \int d\mathbf{q} \frac{V(q)}{2(2\pi)^2} (|J_{11}(\mathbf{q})|^2 + |J_{10}(\mathbf{q})|^2 - \\ &- \int d\mathbf{q} \frac{V(q)}{2(2\pi)^2} (\frac{1}{2}|J_{21}(\mathbf{q})|^2 + \frac{1}{2}|J_{20}(\mathbf{q})|^2) + \\ &+ \int d\mathbf{q} \frac{V(q)}{2(2\pi)^2} \sum_{N=2} |J_{2,N}(\mathbf{q}) J_{0,N-2}^*(\mathbf{q})| \end{aligned} \quad (20)$$

This value may depend on resolving the divergency problem, but it is not zero. There is no Kohn's theorem³⁴.

For the excitations in the different valleys energy splitting due to the layer asymmetry is absent.

The same transitions may occur for the spin ferromagnetic state, where Zeeman splitting would be included: one spin component σ_1 of LLs 0 and 1 is completely filled

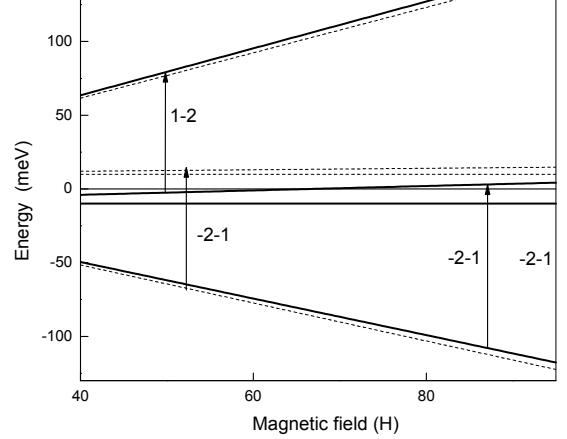


FIG. 1: The one-electron energy levels and cyclotron transitions taking into account the next-nearest-neighbor transitions and layer asymmetry (solid for K valley and dash for K' valley).

and the other σ_2 is completely empty in both valleys. There are two excitations $Q_{1,2;\xi\sigma_1}^+$ and two excitations $Q_{-2,1;\xi,\sigma_2}^+$ with the same energies.

B. The asymmetric bilayer with electron-hole asymmetry.

In this case at the total filling-factor $\nu = 0$ the filling of Landau levels depends of the magnetic field.

If the magnetic field is not so high and

$$\left(\frac{\gamma_4}{\gamma_0} - \frac{\tilde{\Delta}}{2\gamma_1}\right) \hbar\omega_c - \delta < \frac{U}{2} \quad (21)$$

then in the valley with $\xi = -1$ levels 0 and 1 are filled and in the other valley with $\xi = 1$ they are empty. There are cyclotron electron or hole transitions $(1,2)$ from the top filled to the next empty level as for the case A (see Fig.1, left). The noninteracting parts of transition energies are presented by the following expressions:

$$\Delta E_{1,2}(-1) = \hbar\omega_c(\sqrt{2} + \frac{\gamma_4}{\gamma_0} - \frac{\tilde{\Delta}}{2\gamma_1}) + \frac{1}{2}(U - \delta) \quad (22)$$

$$\Delta E_{-2,1}(1) = \hbar\omega_c(\sqrt{2} - \frac{\gamma_4}{\gamma_0} + \frac{\tilde{\Delta}}{2\gamma_1}) + \frac{1}{2}(U - \delta) \quad (23)$$

The Coulomb parts are the same, as it was discussed before. The difference in the energies between the valley transitions is due to the electron-hole asymmetry.

$$\Delta E_{1,2}(-1) - \Delta E_{-2,1}(1) = \left(\frac{2\gamma_4}{\gamma_0} - \frac{\tilde{\Delta}}{\gamma_1}\right) \hbar\omega_c \quad (24)$$

If the magnetic field is sufficiently high and

$$\left(\frac{\gamma_4}{\gamma_0} - \frac{\tilde{\Delta}}{2\gamma_1}\right)\hbar\omega_c - \delta > \frac{U}{2} \quad (25)$$

then in both valleys levels 1 are empty, but the level 0 is filled for the K valley and empty for the K' valley (see Fig.1, right). There are only hole-type transitions $(-2,1)$. The following noninteracting parts of transition energies are

$$\Delta E_{-2,1}(1) = \hbar\omega_c(\sqrt{2} - \frac{\gamma_4}{\gamma_0} + \frac{\tilde{\Delta}}{2\gamma_1}) + \frac{1}{2}(U - \delta) \quad (26)$$

$$\Delta E_{-2,1}(-1) = \hbar\omega_c(\sqrt{2} - \frac{\gamma_4}{\gamma_0} + \frac{\tilde{\Delta}}{2\gamma_1}) - \frac{1}{2}(U - \delta) \quad (27)$$

Because of different filling of Landau levels in the valleys the influence of Coulomb interaction differs in the part of the self-energies.

$$\frac{1}{2}\Sigma_{20} - \Sigma_{10} = \int d\mathbf{q} \frac{V(q)}{(2\pi)^2} (|J_{10}(\mathbf{q})|^2 - \frac{1}{2}|J_{20}(\mathbf{q})|^2) \quad (28)$$

$$\Sigma_{10} - \frac{1}{2}\Sigma_{20} = \frac{7}{16}E_c\sqrt{\frac{\pi}{2}} \quad (29)$$

$$\tilde{E}_{-2,1}(1) - \tilde{E}_{-2,1}(-1) = (U - \delta) + \frac{7}{16}E_c\sqrt{\frac{\pi}{2}} \quad (30)$$

The splitting between valley cyclotron transitions is due to Coulomb interaction and layer asymmetry

C. The bilayer graphene in the full-symmetric case.

In this case labelled h we have two half-filled zero-energy levels in both valleys: 0 ($\nu_0 = 1/2$) and 1 ($\nu_1 = 1/2$); this means $\nu = 2$ for the electrons in each valley and $\nu = 2$ for the holes in each valley. For each valley there are two transitions: $(1, 2)$ from half-filled to empty levels and $(-2, 1)$ from filled to half-empty levels. These transitions are connected by the Coulomb interaction \tilde{V} . Using the Hartree-Fock approximation for non-integer filling-factors^{35,36} two combined modes $Q_{s,a}^+$ with the energies $E_{s,a}$ are found.

$$Q_{s,a}^+ = \frac{1}{\sqrt{2}}(Q_{1,2,\xi,\sigma}^+ \pm Q_{-2,1,\xi,\sigma}^+)$$

$$E_{s,a} = \omega_c\sqrt{2} + E_c^h \pm \frac{1}{2}\tilde{V} \quad (31)$$

These modes may be called symmetric and antisymmetric in analogy to modes in semiconductor bilayer.

$$E_c^h = \frac{1}{2}E_{ex,0I}^{(1,2)} + (\Sigma_2 - \Sigma_1)_h; \quad (\Sigma_2 - \Sigma_1)_h \neq \frac{1}{2}(\Sigma_2 - \Sigma_1)_{0I} \quad (32)$$

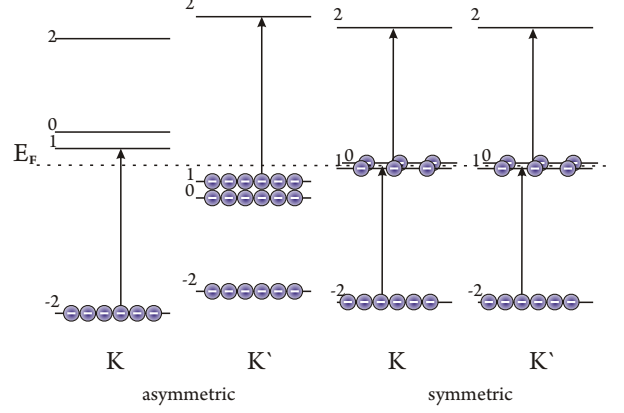


FIG. 2: The one-electron Landau energy levels and cyclotron transitions for symmetric and asymmetric (B) cases. For the symmetric case there are two transitions in each valley.

$$\tilde{V} = V_{-2,1}^{1,2} = V_{1,2}^{-2,1} = \frac{1}{(2\pi)^2} \int d\mathbf{q} V(q) \tilde{J}_{12}(\mathbf{q}) \tilde{J}_{-2,1}(-\mathbf{q}) =$$

$$= \frac{1}{2(2\pi)^2} \int d\mathbf{q} V(q) |J_{12}|^2(\mathbf{q}) = \frac{1}{2} \sqrt{\frac{\pi}{2}} E_c * \frac{7}{16} \quad (33)$$

$$\tilde{V} = 2.5\sqrt{B} \text{ and for } B = 40T \quad \tilde{V} \simeq 15\text{meV}.$$

This splitting for combined electron-hole transitions from half-filled level is not specific to bilayer graphene. In monolayer graphene with valley asymmetry considered in¹⁹ the filling-factors were $\nu = 2$ for electrons in one valley and $\nu = 2$ for holes in another valley. In the simple case with symmetric valleys we have the half-filled zero-energy level in both valleys which means $\nu = 1$ for both electrons and holes in each valley. For monolayer graphene with half-filled zero-energy level there are related transitions $(0, 1)$ for the electrons and $(-1, 0)$ for the holes, and the corresponding value of splitting for the combined modes $\tilde{V}_{mg} = \frac{1}{4}\sqrt{\frac{\pi}{2}}E_c$ is found using the wave functions from¹⁹. This value is nearly the same as for bilayer graphene ($\tilde{V}_{mg} \simeq 2.5\sqrt{B}$), but for monolayer graphene it is possible to observe this splitting for lower experimentally used magnetic fields.

D. The symmetric bilayer with electron-hole asymmetry.

For the bilayer graphene with symmetric layers and therefore the symmetric valleys, but with the electron-hole asymmetry included there are half-filled 0 Landau levels ($\nu_0 = 1/2$) and empty 1 Landau levels ($\nu_1 = 0$) in both valleys. For each valley there are only hole-type $(-2, 1)$ transitions. Coulomb corrections are equal for both valleys due to the same filling, and there is no splitting in this case.

IV. SUMMARY AND DISCUSSION

In conclusion, the cyclotron transitions for clean bilayer graphene are studied. The electron-hole asymmetry, the small layer asymmetry and the influence of Coulomb interaction are taken into account. The charge-density excitations at small momenta are considered within the Hartree-Fock approximation. It is shown that the energies of cyclotron transitions in two valleys can be equal or can be split either due to the electron-hole asymmetry or due to Coulomb interaction. The splitting depends on applied magnetic and electric fields. At lower magnetic fields the energy splitting in the spectrum is due to electron-hole one-particle asymmetry while higher magnetic fields the energy splitting in the spectrum is due to Coulomb interaction. For the fully symmetric case with half-filled zero-energy levels (where electron-hole and layer asymmetries are not taken into account) the energy splitting proportional to Coulomb interaction is found both for bilayer and monolayer graphene.

In the work²⁹ the many-body corrections to cyclotron resonance in monolayer and bilayer graphene due to Coulomb interaction were studied neglecting the electron-hole asymmetry. In the following work³⁰ of the same author the weak electron-hole asymmetry was included, where the cyclotron resonances were considered using the four-band Hamiltonian. The four-band Hamiltonian consideration is more precise, though the results concerning the noninteracting part coincide in the limits of accuracy. The employed SMA method to study

many-body corrections due to Coulomb interaction (single mode approximation) for integer filling-factors is identical to that used in the present work. Unfortunately it is impossible to compare directly the results of these work with ours in the aspect of many-body corrections because in^{29,30} the different filling-factors ($\nu = 4, 8, 12, 16$) were considered.

In the experimental work⁷ graphene bilayers embedded in the multilayer epitaxial graphene were studied. The splitting between electron-type and hole-type transitions is found in a relatively narrow range of B . The possible explanation invoking the electron-hole asymmetry yields the right values, but should be seen for wide range of magnetic fields. Moreover, in that work the other higher energy transitions corresponding to $\Delta n = 1$ were studied. In our notation the experimentally studied transitions are $(-2,3)$ and $(-3,2)$. To compare the experimental results with the theory it is necessary to study these transitions in the future.

In this work there are a simultaneous consideration of both Coulomb interaction and electron-hole asymmetry, and suggestions concerning observation of splitting due to these factors at different regimes.

V. ACKNOWLEDGEMENTS

Support is acknowledged to the Program LIA ENS-Landau, the Russian Foundation of Basic Research, project 10-02-00131, and to the ANR program in France (the project BLAN07-3-192276).

-
- * Electronic address: bisti@issp.ac.ru
- ¹ A.H. Castro Neto, F. Guinea, N.M.R. Peres, K.S. Novoselov and A.K. Geim. *Rev. Mod. Phys.* **81**, 109 (2009)
 - ² M. Mucha-Kruczynski, E. McCann and Vladimir I. Falko. *Semicond. Sci. Technol.* **25**, 033001 (2010)
 - ³ E.A. Henriksen, Z. Jiang, L.-C. Tung, M.E. Schwarz, M. Takita, Y.-J. Wang, P. Kim, and H.L. Stormer. *Phys. Rev. Lett.* **100**, 087403 (2008).
 - ⁴ Z.O. Li, E.A. Henriksen, Z. Jiang, Z. Hao, M.C. Martin, P. Kim, H.L. Stormer and D.N. Basov. *Phys. Rev. Lett.* **102**, 037403 (2009).
 - ⁵ L.M. Zhang, Z.Q. Li, D.N. Basov, M.M. Fogler, Z. Hao and M. C. Martin. *Phys. Rev.* **B78**, 235408 (2008).
 - ⁶ A.B. Kuzmenko, E. van Heumen, D. van der Marel, P. Lerch, P. Blake, K.S. Novoselov, and A.K. Geim. *Phys. Rev.* **B79**, 115441 (2009).
 - ⁷ M. Orlita, C. Faugeras, J. Borisiuk, J.M. Baranowski, W. Strupinski, M. Sprinkle, C. Berger, W.A. de Heer, D.M. Basko, G. Martinez, and M. Potemski. *Phys. Rev.* **B83**, 125302 (2011).
 - ⁸ Edward McCann. *Phys. Rev.* **B74**, 161403 (2006).
 - ⁹ Edward McCann and Vladimir I. Fal'ko. *Phys. Rev. Lett.* **96**, 086805 (2006).
 - ¹⁰ Z. Jiang, E.A. Henriksen, L.-C. Tung, Y.-J. Wang, M.E. Schwarz, M.Y. Han, P. Kim, and H.L. Stormer. *Phys. Rev. Lett.* **98**, 197403 (2007).
 - ¹¹ R.S. Deacon, K.-C. Chuang, R.J. Nicholas, K.S. Novoselov, and A.K. Geim. *Phys. Rev.* **B76**, 081406 (2007).
 - ¹² E.A. Henriksen, P. Cadden-Zimansky, Z. Jiang, Z.O. Li, L.-C. Tung, M.E. Schwarz, M. Takita, Y.-J. Wang, P. Kim, and H.L. Stormer. *Phys. Rev. Lett.* **104**, 067404 (2010).
 - ¹³ Mikito Koshino and Tsuneya Ando. *Phys. Rev.* **B77**, 115313 (2008).
 - ¹⁴ M.L. Sadowski, G. Martinez, and M. Potemski, C. Berger and W.A. de Heer. *Phys. Rev. Lett.* **97**, 266405 (2006).
 - ¹⁵ P. Plochocka, C. Faugeras, M. Orlita, M.L. Sadowski, G. Martinez, and M. Potemski, M.O. Goerbig and J.-N. Fuchs, C. Berger and W.A. de Heer. *Phys. Rev. Lett.* **100**, 087401 (2008).
 - ¹⁶ M. Orlita, C. Faugeras, G. Martinez, D.K. Maude, M.L. Sadowski, and M. Potemski. *Phys. Rev. Lett.* **100**, 136403 (2008).
 - ¹⁷ M. Orlita, C. Faugeras, P. Plochocka, P. Neugebauer, G. Martinez, D.K. Maude, A.-L. Barra, M. Sprinkle, C. Berger, W.A. de Heer, and M. Potemski. *Phys. Rev. Lett.* **101**, 267601 (2008).
 - ¹⁸ Y. Zhao, P. Cadden-Zimansky, Z. Jiang, and P. Kim. *Phys. Rev. Lett.* **104**, 066801 (2010).
 - ¹⁹ Yu.A. Bychkov and G. Martinez. *Phys. Rev.* **B77**, 125417 (2008).
 - ²⁰ A. Iyengar, Jianhui Wang, H.A. Fertig, and L. Brey. *Phys. Rev.* **75**, 125430 (2007).

- ²¹ V.E. Bisti, N.N. Kirova. JETP Lett.**90**, 120 (2009).
- ²² T. Misumi and K. Shizuya. Phys. Rev. **B77**, 195423 (2008).
- ²³ Marcus Mueller and Subir Sachdev. Phys. Rev. **B78**, 115419 (2008).
- ²⁴ R. Roldan, J.-N.Fuchs, and M. O. Goerbig.**B80**, 085408 (2009)
- ²⁵ R. Roldan, J.-N.Fuchs, and M. O. Goerbig.**B82**, 205418 (2010).
- ²⁶ Yafis Barlas, R. Cote, K. Nomura, and A.H. MacDonald. Phys. Rev. Lett. **101**, 097601 (2008).
- ²⁷ Csaba Toke, Vladimir I. Falko. Phys. Rev. **B83**, 115455 (2011).
- ²⁸ K. Shizuya. Phys. Rev. **B79**, 165402 (2009).
- ²⁹ K. Shizuya. Phys. Rev. **B 81**, 075407 (2010).
- ³⁰ K. Shizuya. Phys. Rev. **B 84**, 075409 (2011).
- ³¹ J. Milton Pereira, Jr., F.M. Peeters, and P. Vasilopoulos. Phys. Rev. **B76**,115419 (2007).
- ³² Y.H. Lai, J.H. Ho, C.P. Chang, and M.F. Lin. Phys. Rev. **B77**,085426 (2008).
- ³³ C. Kallin and B. I. Halperin, Phys. Rev. B 30, 5655 (1984).
- ³⁴ D.S.L. Abergel and Vladimir Fal'ko. Phys. Rev. **B75**, 155430 (2007).
- ³⁵ V.E. Bisti. JETP Letters 69, 584,(1999).
- ³⁶ Yu.A. Bychkov and G. Martinez. Phys. Rev. **B66**,193312 (2002).